Disposition Effect and Multi-Asset Market Dynamics

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Disposition effect and multi-asset market dynamics

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Abstract

Purpose – Asset pricing dynamics in a multi-asset framework when investors’ trading exhibits the disposition effect is studied. The purpose of this paper is to explore asset pricing dynamics and the switching behavior among multiple assets.

Design/methodology/approach – The dynamics of complex financial markets can be best explored by following agent-based modeling approach. The artificial financial market is populated with traders following two heterogeneous trading strategies: the technical and the fundamental trading rules. By simulation, the switching behavior among multiple assets is investigated.

Findings – The proposed framework can explain important stylized facts in financial time series, such as random walk price dynamics, bubbles and crashes, fat-tailed return distributions, absence of autocorrelation in raw returns, persistent long memory of volatility, excess volatility, volatility clustering and power-law tails. In addition, asset returns possess fractal structure and self-similarity features; though the switching behavior is only allowed among the asset markets.

Practical implications – The model demonstrates stylized facts of most real financial markets. Thereafter, the proposed model can serve as a testbed for policy makers, scholars and investors.

Originality/value – To the best of knowledge, no research has been conducted to introduce the disposition effect to a multi-asset agent-based model.

Keywords Agent-based model, Financial markets, Bounded rationality, Fractals and scaling, Power-law distributions

Paper type Research paper

1. Introduction

Shefrin and Statman (1985) provide the first key analysis of the disposition effect. Empirical evidence shows that investors tend to sell the winning assets too soon and ride the losing ones too long. This behavioral bias leads to bounded rationality, where decision-making process is affected by beliefs and intuitions rather than rational considerations (Gärling et al., 2017). Agent-based modeling provides a promising approach for modeling heterogeneous, bounded rationality behaviors. Many models have been developed to explain the complex behavior of financial markets (cf. Day and Huang, 1990; Chiarella, 1992; Föllmer and Schweizer, 1993; Brock and LeBaron, 1995; Brock, 1996; Brock and Hommes, 1998; Hommes, 2001; Farmer and Joshi, 2002; Westerhoff, 2008; Lux, 2009; Jongen et al., 2012; Šperka and Spišák, 2012; Chia et al., 2014; Kirman, 2014). These studies suggest that artificial financial markets populated with heterogeneous beliefs can explain many stylized facts observed empirically (cf. Mandelbrot, 1983; Fama, 1970; Cont, 2001, 2005). Stylized facts represent a set of statistical properties common across many markets, such as bubbles and crashes, fat-tailed return distributions, uncorrelated returns and volatility clustering (cf. Cont, 2001, 2005; Gabaix et al., 2003; Gilli and Kellezi, 2006; Lux, 2006; Haas and Bigorsch, 2011). In addition, Lux and Marchesi (1999) find that scaling in price changes results from the trading process itself.

The above researches are concerned with studying one risky market only. Few efforts have been spent to investigate asset pricing dynamics in a multi-asset framework, such as Westerhoff (2004) and Chiarella et al. (2007). Moreover, the interest of introducing behavioral trading biases into agent-based frameworks is increasing (cf. Takahashi and Terano, 2003; Lovric et al., 2010; Kukacka and Barunik, 2013; Selim et al., 2015; Feldman and Lepori, 2016). Li et al. (2014) introduce the disposition effect to the framework used by...
Chiarella (1992) and Westerhoff (2008). Furthermore, Ezzat (2016) introduce loss-aversion behavioral bias to the multi-asset framework of Westerhoff (2004). Loss averse traders realize their losses more strongly than their perception of equivalent gains. This anomaly of perceiving losses may lead traders to hold the losing assets too long, which is the disposition effect bias. This paper introduces the disposition effect to the multi-asset framework proposed by Westerhoff (2004). The behavioral model aims at exploring asset pricing dynamics and the switching behavior among multiple assets. To the best of knowledge, no research has been conducted to introduce the disposition effect to a multi-asset agent-based model.

Previous studies reveal that most traders rely on two trading philosophies: the technical and the fundamental trading (cf. Taylor and Allen, 1992; Menkhoff, 1997; Frankel and Froot, 1987a, b, 1990). Therefore, the proposed artificial financial market is populated with traders following two heterogeneous trading beliefs practically expressed as technical and fundamental prediction rules. Technical traders (or chartists) believe in price trend continuation and benefit from price changes (Murphy, 1999). On the other hand, fundamentalists expect that asset price will revert to its fundamental value in the short run (Graham and Dodd, 2009). Each trader perceives asset prices and submits an order. Note that the main purpose of the model is to imitate the agents’ switching behavior among multiple assets. The switching behavior between trading strategies is not the main concern of this model. Thereafter, fundamentalists are restricted to trade in only one asset. Thus, the fraction of fundamentalists who are trading this asset is fixed. Conversely, chartists can switch among asset markets but they are restricted to trade only one asset at a time, thus, they do not build portfolios with several risky positions. The switching behavior among asset markets is mainly affected by the disposition effect. If the price of an asset decreases for less than some reference points, agents following technical analysis will keep holding this asset and switch their orders to the other asset markets. The fraction of each asset to be traded depends on the payoff, or attractiveness, of that asset and on its past performance, which exemplifies the learning behavior. The orders are sent in forms of messages to the market makers who settle assets’ prices according to a stated price adjustment rule. Financial assets are priced according to the net-submitted orders, where buying (excess demand) drives the log price up and selling (excess supply) drives it down. Once the new price is declared, each agent decides on which action to take: buy, sell or hold according to its trading strategy. Due to the disposition effect, chartists hang onto losing assets for a long time.

The proposed model can replicate important stylized facts, such as random walk prices, bubbles and market crashes, fat-tailed returns, power-law tails, long memory in volatility, the mean-reversion behavior of returns, long-range power-law autocorrelations in absolute returns. In addition, asset returns possess fractal structure and self-similarity features.

The rest of the paper is organized as follows. In Section 2, the proposed artificial financial market model is presented. Section 3 is devoted to exploring asset pricing dynamics and presenting the results of extensive Monte Carlo simulation. The last section concludes the paper.

2. The model
There are $k$, $k = 1, 2, \ldots, K$, assets to be traded. Trading takes place at discrete time steps $t$, $t = 1, \ldots, T-1$, the time periods are interpreted as “trading days.” It is assumed that the assets are traded in symmetric markets. Each trader can hold a fractional share, which is a share of equity that is less than one full share, and the trader can accumulate an unbounded inventory. Agents in the artificial market interact indirectly through their impact on the
price adjustment of asset $k$, $k = 1, 2, \ldots, K$, which affects the chartists’ decision to switch among the multiple assets.

There are three types of artificial agents in the market: technical traders, fundamental traders and market makers. The chartists conduct technical analysis to form their expectation of future asset log prices. Chartists form their expectation based on the most basic form of momentum rule AR(1) (Jongen et al., 2012):

$$E^c_t(p^k_{t+1}) = p^k_t + \hat{b}(p^k_t - p^k_{t-1}),$$

where $E^c_t(p^k_{t+1})$ is the next period’s expected log prices using technical trading, $p^k_t$ is the log price of asset $k$ at time $t$, $\hat{b} > 0$ measures the rate at which chartists extrapolate the past into the future. The second term between parentheses at the right-hand side of (1) represents the difference between current and last log prices, which reveals the exploitation of price changes. Similar expectation formations are, for instance, used by Day and Huang (1990), Föllmer and Schweizer (1993), Brock and Hommes (1998), Hommes (2001), Farmer and Joshi (2002), Westerhoff (2004, 2008), Jongen et al. (2012), Sperka and Spišák (2012), Chia et al. (2014), Kirman (2014), Li et al. (2014), Selim et al. (2015) and Ezzat (2016).

However, chartists’ trading exhibits the behavioral bias of disposition effect. This can be expressed in terms of orders exploiting technical trading rule as inspired by Li et al. (2014). Thus, technical trading rule of chartist, $D^c_t$, at time $t$ is given as follows:

$$D^c_t = \begin{cases} 0 & \text{if } p^k_t - p^k_{t-1} < \theta \\ \hat{b} [E^c_t(p^k_{t+1}) - p^k_t] + \beta^k_t & \text{else} \end{cases},$$

where $\hat{b} > 0$ is a positive reaction parameter that captures the strength of agents’ sensitivity to price signals. By substituting (1) in (2), the demand of the chartists is given by:

$$D^c_t = \begin{cases} 0 & \text{if } p^k_t - p^k_{t-1} < \theta \\ b(p^k_t - p^k_{t-1}) + \beta^k_t & \text{else} \end{cases},$$

where $b = \hat{b}\bar{b}$ is the extrapolation parameter (cf. Föllmer and Schweizer, 1993; Jongen et al., 2012; Chia et al., 2014), $\theta < 0$ is the reference point, $\beta^k_t$, $t = 1, \ldots, T$ are IID normally distributed random variables with mean zero and constant standard deviation $\sigma_{\beta^k}$. The random term, $\beta^k_t$, is added to capture random chartist orders. $D^c_t < 0$ indicates selling orders and $D^c_t > 0$ indicates buying orders. If the price of asset $k$ falls less than some reference points, $\theta$, chartists will continue to hold the losing assets and set their demand of asset $k$ to 0. Otherwise, chartists will follow the trend.

Regressive expectations of fundamentalists can be expressed by:

$$E^f_t(p^k_{t+1}) = p^k_t + \hat{c}(f^k_t - p^k_t),$$

where $E^f_t(p^k_{t+1})$ is the next period’s expected log prices following the fundamental trading rule, $f^k_t$ is the log fundamental value of asset $k$ at time $t$, and $\hat{c} > 0$ is the expected adjustment speed of the asset log price toward its log fundamental value. It is assumed that traders can calculate the fundamental values. The second term between parentheses at the right-hand
side of (4) represents the deviation of asset log prices from their log fundamentals, known as market distortion:

\[
\text{dist}^k_t = f^k_t - p^k_t.
\] (5)

A positive value of \(\text{dist}^k_t\) shows a bubble, while a negative value means that the asset is undervalued at time \(t\). Similar expectation formations are, for instance, used by Day and Huang (1990), Föllmer and Schweizer (1993), Brock and Hommes (1998), Hommes (2001), Winker and Gilli (2001), Farmer and Joshi (2002), Westerhoff (2004, 2008), Jongen et al. (2012), Šperka and Spišák (2012), Chia et al. (2014), Kirman (2014), Li et al. (2014), Selim et al. (2015) and Ezzat (2016).

Henceforth, orders generated by fundamental trading rule, \(D^f_{k}^t\), at time \(t\) can be written as:

\[
D^f_{k}^t = \hat{c}E^f_{k}^t \left(f^k_t - p^k_t\right) + \gamma^k_t,
\] (6)

where \(\hat{c} > 0\) is a positive reaction parameter that captures the sensitivity of fundamentalists’ excess demand to deviations of the log price from the underlying log fundamental value.

By substituting (4) in (6), the demand of the fundamentalists is given by:

\[
D^f_{k}^t = \tilde{c} \left(f^k_t - p^k_t\right) + \gamma^k_t,
\] (7)

where \(\tilde{c} = \hat{c}\tilde{c}\) is the reverting parameter (cf. Föllmer and Schweizer, 1993; Jongen et al., 2012; Chia et al., 2014). The random term \(\gamma^k_t\) is introduced to capture additional random orders of fundamentalists. \(\gamma^k_t, \ t = 1, \ldots, T\) are IID normally distributed random variables with mean zero and constant standard deviation \(\sigma_{\gamma^k}\). \(D^f_{k}^t < 0\) indicates selling orders, while \(D^f_{k}^t > 0\) indicates buying orders.

The log fundamental values are assumed to evolve following a random walk such that:

\[
f^k_t = f^k_{t-1} + \eta^k_t,
\] (8)

where \(f^k_t\) and \(\eta^k_t\) are log fundamental values and fundamental shock values, respectively, and \(\eta^k_t, \ t = 1, \ldots, T-1\) are IID normally distributed with mean zero and standard deviation \(\sigma_{\eta^k}\). For simplicity, it is assumed that the assets’ fundamental volatility is equal for all the assets. Also, it is assumed that traders can calculate the fundamental values.

The market makers are risk neutral and settle assets’ prices according to the aggregate excess demand (or net orders) based on a price settlement rule which can be described as:

\[
p^k_{t+1} = p^k_t + aD^f_{t+1} + x^k_t,
\] (9)

where \(a\) is a positive price settlement parameter (cf. Chia et al., 2014; Kirman, 2014), \(D^f_{t+1}\) is the aggregate excess demand, and the noise term \(x^k_t\) is added to catch any random factors affecting the price settlement process. It is assumed that \(x^k_t, \ t = 1, \ldots, T\) are IID normally distributed random variables with mean zero and constant standard deviation \(\sigma_{x^k}\). Similar expectation formations are, for instance, used by Farmer and Joshi (2002),
According to (9), prices change mainly depending on the excess demand, i.e.:

\[ p_{t+1}^k - p_t^k = aD_t^k + z_t^k, \]  

(10)

where \( D_t^k \) is calculated as follows:

\[ D_t^k = w_t^k D_t^{k'} + D_t^{k,h}, \]  

(11)

where \( D_t^{k'} \) and \( D_t^{k,h} \) are orders submitted by chartists and fundamentalists, respectively, at time \( t \) and \( w_t^k \) is the weight of asset \( k \) traded at time \( t \). The details of computing the weights, \( w_t^k \), are provided shortly.

The evolutionary part of the model depicts how assets are selected over time. The evolutionary fitness measure (or attractiveness of the asset) can be presented as follows:

\[ A_t^k = \log \left[ \frac{1}{1 + d|f_t^k - p_t^k|} \right] + mA_{t-1}^k, \]  

(12)

where \( d > 0 \) and \( 0 \leq m < 1 \) are fitness parameter and memory parameter, respectively.

The fitness measure is bounded between \( -\infty \) and \( 0 \) and reaches its maximum value \( 0 \), when \( f_t^k = p_t^k \). Therefore, the larger the distance between \( f_t^k \) and \( p_t^k \), defined by the distortion, the lower the fitness of the asset for chartists.

The evolutionary fitness measure is mirrored in the fraction, \( w_t^k \), that represents the portion of asset \( k \) traded at time \( t \); and all asset weights add up to one. Following Brock and Hommes (1998), the fraction of asset \( k \) traded at time \( t \) is given by the discrete-choice model proposed by Manski and McFadden (1981):

\[ w_t^k = \frac{\exp \left( rA_t^k \right)}{\sum_{k=1}^K \exp \left( rA_t^k \right)}, \]  

(13)

where \( r \geq 0 \) is the intensity of choice parameter and measures the chartists’ sensitivity to choose the most attractive asset. An increase in the intensity of choice can be interpreted as an increase in the rationality of the traders. For \( r = 0 \), the chartists do not realize any differences in the fitness of different assets. In this case, chartists will split their trading between assets equally. If \( r \) goes to infinity, all chartists trade the asset with the highest fitness. The higher the attractiveness of asset \( k \), the more chartists will trade that asset.

3. Simulation results and analyses

3.1 Simulation Design

Unfortunately, the complexity of the model precludes estimating its parameters (cf. Farmer and Joshi, 2002; Winker and Gilli, 2001; Westerhoff, 2004, 2008; Li et al., 2014). The key parameters of the model are the reaction parameters of chartists and fundamentalists, \( b \) and \( c \),
respectively, and the price adjustment parameter, \( a \). Empirical studies based on daily data indicate that the reaction parameters of technical and fundamental trading rules (multiplied with the price adjustment parameter) range between 0 and 0.1 (Westerhoff, 2008). As chartists are the ones who can switch between the asset markets, the extrapolating parameter, \( b \), is set much higher than the reverting parameter, \( c \). Thereby, the adjustment parameter, \( a \), is set to the value of 1. The memory parameter, \( m \), represents the contribution of past realized payoff of choosing this asset in the current trading step so \( m \) is set to the value of 0.5. The values of the rest of the parameters are chosen so that the model can mimic the dynamics of real financial markets. To implement the proposed artificial financial market, an agent-based simulation model is developed using NetLogo platform (Wilensky, 1999). NetLogo provides an environment for simulating natural and social phenomena. It is particularly well suited for modeling complex systems evolving over time. The simulation framework is implemented by using the parameter setting illustrated in Table I. The performance of 5,000 daily observations is investigated. In the following section, simulation results are displayed.

3.2 Stylized facts replication
In this section, the model is validated by investigating the extent to which it is able to replicate the stylized facts observed empirically. Before turning to a comprehensive Monte Carlo simulation, first a representative simulation run will be explored. Figures 1–8 depict the dynamics of the representative simulation. Figure 1 illustrates the evolution of the five assets’ log prices. Note that prices fluctuate around their fundamental values. The average distortions \( \text{dist} \), where \( \text{dist} = \sum_{t=1}^{T} |\text{dist}_t| \), are estimated as 11.34, 11.61, 10.59, 11.34 and 12.86 percent for the five asset markets, respectively. In addition, all price series are random walk and show bubbles and crashes (the variance-ratio test reports \( p \)-values as 0.15, 0.78, 0.31, 0.22 and 0.29 for the five log prices, respectively. Thereafter, the variance-ratio test does not reject the random walk hypothesis at the 5 percent level). These results are in accordance with the stylized facts that are observed in real financial markets.

Figure 2 displays the returns of the five asset markets. It can be observed that the minimum(maximum) returns are −15(17), −13(15), −16(14), −16(17) and −17(19) percent for all five assets, respectively. Guillaume et al. (1997) is followed to calculate the average absolute returns (\( \text{vol} \)) by; \( \text{vol} = \frac{1}{T} \sum_{t=1}^{T} |r_t| \), which are computed as 1.33, 1.41, 1.59, 1.55 and 1.41 percent for the five asset markets, respectively. These figures indicate excess volatility feature. As stated by Shiller (1981), such volatility values are too large to be explained by

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Description of parameter</th>
</tr>
</thead>
<tbody>
<tr>
<td>( a )</td>
<td>1</td>
<td>Price settlement parameter</td>
</tr>
<tr>
<td>( b )</td>
<td>0.1</td>
<td>Extrapolating parameter</td>
</tr>
<tr>
<td>( c )</td>
<td>0.02</td>
<td>Reverting parameter</td>
</tr>
<tr>
<td>( d )</td>
<td>0.25</td>
<td>Fitness parameter</td>
</tr>
<tr>
<td>( r )</td>
<td>300</td>
<td>Intensity of choice parameter</td>
</tr>
<tr>
<td>( m )</td>
<td>0.5</td>
<td>Memory parameter</td>
</tr>
<tr>
<td>( \theta )</td>
<td>−0.05</td>
<td>Disposition effect parameter</td>
</tr>
<tr>
<td>( \sigma_{w}^f )</td>
<td>0.01</td>
<td>Standard deviation of the fundamental shocks</td>
</tr>
<tr>
<td>( \sigma_{w}^p )</td>
<td>0.05</td>
<td>Standard deviation of the random factors affecting the price settlement process</td>
</tr>
<tr>
<td>( \sigma_{w}^t )</td>
<td>0.01</td>
<td>Standard deviation of the additional random orders of technical trading</td>
</tr>
<tr>
<td>( \sigma_{w}^f )</td>
<td>0.01</td>
<td>Standard deviation of the additional random orders of fundamental trading</td>
</tr>
</tbody>
</table>

Table I. Parameters for the simulation of the multi-asset artificial financial market model
fundamental crashes alone. Furthermore, Figure 2 displays another stylized fact of asset markets, which is volatility clustering.

Figure 3 displays the weight of assets traded by the chartists. The average weights are computed as 9.92, 18.34, 15.87, 23.83 and 32.01 percent for the five assets, respectively. This implies that trading asset $k = 5$ is more profitable. The statistical mean, $\bar{x}$, of the Monte Carlo simulation for each statistic is computed as $\bar{x} = (1/N) \sum_{i=1}^{N} x_i$, where $x_i$, $i = 1, \ldots, N$ are the statistics calculated for each simulation run. Accordingly, the results of 1,000 simulation runs reveal that the mean of average weights is 19.99, 20.30, 19.60, 19.85 and 19.92 percent for the five assets, respectively. The weights of traded assets are almost the same. Therefore, there is no particular asset market that dominates the others.
Another important stylized fact is power-law tails, with a tail index somewhere in the region 2–5. The exponent $\alpha$ of the Pareto distribution for the tails is defined by the following inverse cubic law:

$$\text{Prob}(|r| > x) \sim x^{-\alpha}$$

(14)

with $\alpha \approx 3$ (cf. Lux and Marchesi, 1999; Gilli and Kellezi, 2006; Lux, 2009; Haas and Bigorsch, 2011). To investigate the ability of the proposed model to generate return time series that exhibit power-law tails, the Hill-index tail estimate is computed (Hill, 1975). For this purpose, Huisman et al. (2001) are followed to estimate Hill-index tail for the smallest and largest 10 percent of the observations as illustrated in Figure 4. Regression of the smallest (largest) 10 percent of the observations yields a value of $2.44 \pm 0.042$ ($2.67 \pm 0.029$) for $k = 1$, a value of $3.21 \pm 0.039$ ($2.96 \pm 0.036$) for $k = 2$, a value of $2.84 \pm 0.030$ ($2.96 \pm 0.036$) for $k = 3$, a value of $2.44 \pm 0.042$ ($2.67 \pm 0.029$) for $k = 4$ and a value of $3.94 \pm 0.067$ ($3.63 \pm 0.052$) for $k = 5$. These results are in good agreement with the inverse cubic law of (14).
Another important stylized fact to be checked is the autocorrelation in raw returns. Figure 5 depicts the autocorrelations for the first 100 lags of raw returns for the asset markets. The dashed lines present 95% confidence bands according to the assumption of a white-noise process. The raw returns for the five series display autocorrelation coefficients that are not significant over 100 lags meaning that future prices cannot be predicted.

To investigate the predictability of the assets’ volatilities, autocorrelation in absolute returns is studied. Figure 6 displays the autocorrelations for the first 100 lags of absolute returns for the five assets. The dashed lines present 95% confidence bands according to the assumption of a white-noise process. The figure shows that there is a significant positive autocorrelation in absolute returns of the asset markets for up to 15 lags. This implies that volatility can be partially predicted (in the absence of significant predictability of the direction of price movements).
Another important fact of stock markets to be checked is fractality or self-similarity (Mandelbrot, 1983). One way to quantify the self-similarity property is to estimate the so-called scaling exponent (also called self-similarity parameter). For this purpose, detrended fluctuation analysis is performed following Peng et al. (1994). Linear relationship on a log-log scale between the average fluctuation $F_n$ and the time scale, $n$, demonstrates the scaling power law in return distribution. Figure 7 displays the estimation of the scaling exponent, $H_r$, for raw returns of the five assets. A value of $H = 0.5$ corresponds to a white-noise process, a value of $0.5 < H < 1$ indicates long-range power-law

Note: The panels show the estimates of the smallest 10 percent observations (on the left-hand sides) and the largest 10 percent observations (on the right-hand sides)
autocorrelations, and a value of $0 < H < 0.5$ indicates that large and small changes of the time series are more likely to alternate.

The scaling exponent $H_r$ yields a value of $0.48 \pm 0.031$ for $k = 1$, a value of $0.41 \pm 0.072$ for $k = 2$, a value of $0.44 \pm 0.058$ for $k = 3$, a value of $0.48 \pm 0.032$ for $k = 4$ and a value of $0.40 \pm 0.059$ for $k = 5$. Therefore, estimated values of the scaling exponent indicate white-noise processes.

Figure 8 presents the estimation of the scaling exponent, $H_{r|s}$, for absolute returns of the five assets. The scaling exponent $H_{r|s}$ yields a value of $0.57 \pm 0.050$ for $k = 1$, a value of $0.68 \pm 0.058$ for $k = 2$, a value of $0.63 \pm 0.054$ for $k = 3$, a value of $0.57 \pm 0.050$ for $k = 4$ and a value of $0.66 \pm 0.027$ for $k = 5$. Estimated values of the scaling exponent for the five assets indicate long-range power-law autocorrelations in absolute returns.

To understand the model dynamics, two setup runs are executed. First, the simulation is executed under the assumption of only one asset, that is $k = 1$. Figure 9 shows asset log prices and returns using the same parameter settings of Table I and the same random seeds of the simulation run depicted in Figures 1 and 2. The first panel in Figure 9 depicts that prices are highly deviated from their fundamentals. The computed average distortion

Figure 5. Autocorrelation in raw returns of the asset markets
yields a value of 19.65 percent, which is much higher than the average distortions in the five asset markets. In this case, asset prices lack the ability to follow their fundamentals showing market inefficiency. The second panel in Figure 9 displays asset returns that lack the stylized fact of volatility clustering. This is explained by the nonexistence of the switching behavior between the multiple assets. Additionally, asset returns show high excess volatility. The average volatility is estimated as 3.74 percent. This value is much higher than the average volatility observed empirically. Thereby, multi-asset environment improves market efficiency and stability.

Second, the model is executed without the disposition effect behavioral bias, that is (3) is replaced with the following equation:

\[ D_{t}^{k} = b(p_{t}^{k} - p_{t-1}^{k}) + p_{t}^{k}. \]

(15)

This setup explains the contribution of disposition effect to the model dynamics. Figures 10 and 11 show the log prices and asset returns using the same random seeds of Figures 1 and 2,
the parameter settings of Table I and (3) is replaced with (15). Figure 10 shows that prices fluctuate around their fundamental values. However, average distortions are estimated as 11.73, 12.75, 10.80, 11.73 and 14.20 percent for the five asset markets, respectively. These figures are higher than their correspondences of Figure 1. This indicates that disposition effect improves market efficiency by reducing price distortion.

Furthermore, returns illustrated in Figure 11 display excess volatility and volatility clustering fats. Nevertheless, the average volatilities are computed as 1.53, 1.56, 1.56, 1.53 and 1.97 percent for the five asset markets, respectively. Again, these figures are much higher than their correspondence of Figure 2. This implies that disposition effect enhances the market stability.

Summing up, features of the simulation run displayed in Figures 1–8 resemble the behavior observed from the real markets remarkably well. Next, the robustness of these

**Figure 7.**
Estimation of self-similarity parameter for raw returns of the assets

Notes: The slope of the line relating $\log(F(n))$ to $\log(n)$ is the estimated scaling exponent, $n = \{2^3, \ldots, 2^{10}\}$. 

The parameter settings of Table I and (3) is replaced with (15). Figure 10 shows that prices fluctuate around their fundamental values. However, average distortions are estimated as 11.73, 12.75, 10.80, 11.73 and 14.20 percent for the five asset markets, respectively. These figures are higher than their correspondences of Figure 1. This indicates that disposition effect improves market efficiency by reducing price distortion.

Furthermore, returns illustrated in Figure 11 display excess volatility and volatility clustering fats. Nevertheless, the average volatilities are computed as 1.53, 1.56, 1.56, 1.53 and 1.97 percent for the five asset markets, respectively. Again, these figures are much higher than their correspondence of Figure 2. This implies that disposition effect enhances the market stability.

Summing up, features of the simulation run displayed in Figures 1–8 resemble the behavior observed from the real markets remarkably well. Next, the robustness of these
results by performing a comprehensive Monte Carlo analysis is checked. The analysis rests on 1,000 simulation runs, each containing 5,000 observations. All simulation runs are based on the same parameter settings (presented in Table I) with different seeds of the random variables.

Table II reports estimates of the mean and the median of the mean, maximum, minimum, standard deviation, skewness and kurtosis for the five assets. Note the estimates of the mean and the median of the kurtosis, the standardized fourth moment, for the five assets, are all greater than 3, which are higher than that of the normal distribution.

Table III reports estimates of the mean and the median of the Hill tail-index estimators $\hat{\alpha}_k$ for $k \in \{2.5, 5, 10\}$ percent of the smallest (left-tail) and largest (right-tail) returns for the five assets. The results reveal that average Hill tail-index estimates of the largest and
The smallest 10 percent observations are in line with a universal cubic law \((\alpha \approx 3)\) as proposed in the relevant literature. For instance, considering the smallest (largest) 5 percent of observations of \(r_3\) estimate of the tail index reveals a value of 3.34 (2.53) for the median (Table III).

To continue investigating the robustness of the results, Table IV displays estimates of the mean and the median of the autocorrelation of raw returns, \(AC_{\ell}^r\), for lags \(\ell \in \{1, 2, 3\}\), and the autocorrelation of absolute returns, \(AC_{|r|}^\ell\), for lags \(\ell \in \{1, 20, 50, 100\}\). Estimates of the median of the autocorrelation coefficients \(AC_{\ell}^r\) reveal that price increments are mainly uncorrelated. This is in accordance with most real financial markets, as future prices cannot be predicted. On the other hand, estimate of the median of autocorrelation coefficients \(AC_{|r|}^\ell\) reveals a value of 0.11 for \(k=1\), a value of 0.11 for \(k=2\), a value of 0.11 for \(k=3\), a value of 0.11 for \(k=4\) and a value of 0.11 for \(k=5\) for the first lag indicating persistence in volatility.

Finally, the robustness of the scaling power law is checked. Table V displays estimates of the mean and the median for the scaling exponent of raw returns, \(H_r\), and the scaling exponent of absolute returns, \(H_{|r|}\). Estimate of the median of \(H_r\) reveals a value of 0.42 for \(k=1\), a value of 0.42 for \(k=2\), a value of 0.42 for \(k=3\), a value of 0.42 for \(k=4\) and a value of 0.42 for \(k=5\). This implies a small degree for predicting price changes. Moreover, estimate of the median of \(H_{|r|}\) reveals a value of 0.63 for \(k=1\), a value of 0.64 for \(k=2\), a value of 0.63 for \(k=3\), a value of 0.63 for \(k=4\) and a value of 0.63 for \(k=5\). These values show long-range power-law autocorrelations in absolute returns.

4. Conclusion
A multi-asset agent-based model when investors’ trading exhibits the disposition effect is developed to simulate the switching behavior among multiple assets. The disposition effect is one of the most robust behavioral anomalies documented in the literature of behavioral finance. The model is validated by investigating the degree to which it is able to replicate the stylized facts observed empirically. The comprehensive Monte Carlo simulation results reveal that the proposed multi-asset model has a remarkable ability to replicate important stylized facts, such as random walk prices, bubbles and market crashes, fat-tailed returns, power-law tails, long memory in volatility, and long-range
power-law autocorrelations in absolute returns. Moreover, asset returns possess fractal structure and self-similarity features. The fractal hypothesis proposed by Peters (1994) provides an explanation of the fractal structure in financial market. The hypothesis reflects fractality as a result of the switching behavior between short-term and long-term trading. In this paper, the switching behavior between strategies is not allowed though the model exhibits fractal structure and self-similarity. Thereafter, the hypothesis needs to be revised by considering other switching behaviors, i.e. between multiple assets, as one of the main sources of fractality. No doubt that the complex behavior of the financial market precludes the full insight of financial market dynamics.
Table II.
Descriptive statistics for asset returns of the five asset markets

<table>
<thead>
<tr>
<th>Asset</th>
<th>Mean/Median</th>
<th>Mean</th>
<th>Min.</th>
<th>Max.</th>
<th>SD</th>
<th>Skew.</th>
<th>Kurt.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$r_1$</td>
<td>Mean</td>
<td>$-7.76 \times 10^{-7}$</td>
<td>$-0.15$</td>
<td>$0.15$</td>
<td>$0.02$</td>
<td>$0.04$</td>
<td>$5.35$</td>
</tr>
<tr>
<td></td>
<td>Median</td>
<td>$-5.68 \times 10^{-6}$</td>
<td>$-0.15$</td>
<td>$0.15$</td>
<td>$0.02$</td>
<td>$0.04$</td>
<td>$5.38$</td>
</tr>
<tr>
<td>$r_2$</td>
<td>Mean</td>
<td>$5.82 \times 10^{-6}$</td>
<td>$-0.15$</td>
<td>$0.15$</td>
<td>$0.02$</td>
<td>$0.04$</td>
<td>$5.40$</td>
</tr>
<tr>
<td></td>
<td>Median</td>
<td>$6.96 \times 10^{-6}$</td>
<td>$-0.15$</td>
<td>$0.15$</td>
<td>$0.02$</td>
<td>$0.04$</td>
<td>$5.45$</td>
</tr>
<tr>
<td>$r_3$</td>
<td>Mean</td>
<td>$2.74 \times 10^{-7}$</td>
<td>$-0.15$</td>
<td>$0.15$</td>
<td>$0.02$</td>
<td>$0.05$</td>
<td>$5.40$</td>
</tr>
<tr>
<td></td>
<td>Median</td>
<td>$-2.3 \times 10^{-7}$</td>
<td>$-0.15$</td>
<td>$0.15$</td>
<td>$0.02$</td>
<td>$0.06$</td>
<td>$5.42$</td>
</tr>
<tr>
<td>$r_4$</td>
<td>Mean</td>
<td>$-2.6 \times 10^{-6}$</td>
<td>$-0.15$</td>
<td>$0.15$</td>
<td>$0.02$</td>
<td>$0.04$</td>
<td>$5.36$</td>
</tr>
<tr>
<td></td>
<td>Median</td>
<td>$-1.8 \times 10^{-6}$</td>
<td>$-0.15$</td>
<td>$0.15$</td>
<td>$0.02$</td>
<td>$0.04$</td>
<td>$5.42$</td>
</tr>
<tr>
<td>$r_5$</td>
<td>Mean</td>
<td>$8.67 \times 10^{-7}$</td>
<td>$-0.15$</td>
<td>$0.15$</td>
<td>$0.02$</td>
<td>$0.04$</td>
<td>$5.37$</td>
</tr>
<tr>
<td></td>
<td>Median</td>
<td>$-4.2 \times 10^{-7}$</td>
<td>$-0.15$</td>
<td>$0.15$</td>
<td>$0.02$</td>
<td>$0.04$</td>
<td>$5.37$</td>
</tr>
</tbody>
</table>

Notes: The table reports estimates of the mean and the median of the mean, maximum, minimum, SD, skewness and kurtosis. Computations are based on 1,000 time series, each containing 5,000 observations.
### Table III.
The Hill tail-index estimator $\hat{\alpha}_k$ for the left and right tails

<table>
<thead>
<tr>
<th>Asset</th>
<th>Mean/Median</th>
<th>$\hat{\alpha}_{2.5%}$</th>
<th>$\hat{\alpha}_{5%}$</th>
<th>$\hat{\alpha}_{10%}$</th>
<th>$\hat{\alpha}_{2.5%}$</th>
<th>$\hat{\alpha}_{5%}$</th>
<th>$\hat{\alpha}_{10%}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$r_1$</td>
<td>Mean</td>
<td>3.33</td>
<td>3.35</td>
<td>3.11</td>
<td>2.58</td>
<td>2.62</td>
<td>3.22</td>
</tr>
<tr>
<td></td>
<td>Median</td>
<td>3.32</td>
<td>3.30</td>
<td>2.99</td>
<td>2.51</td>
<td>2.53</td>
<td>3.10</td>
</tr>
<tr>
<td>$r_2$</td>
<td>Mean</td>
<td>3.36</td>
<td>3.37</td>
<td>3.11</td>
<td>2.58</td>
<td>2.63</td>
<td>3.25</td>
</tr>
<tr>
<td></td>
<td>Median</td>
<td>3.34</td>
<td>3.31</td>
<td>3.00</td>
<td>2.51</td>
<td>2.56</td>
<td>3.12</td>
</tr>
<tr>
<td>$r_3$</td>
<td>Mean</td>
<td>3.35</td>
<td>3.35</td>
<td>3.10</td>
<td>2.57</td>
<td>2.61</td>
<td>3.21</td>
</tr>
<tr>
<td></td>
<td>Median</td>
<td>3.36</td>
<td>3.34</td>
<td>3.01</td>
<td>2.50</td>
<td>2.53</td>
<td>3.13</td>
</tr>
<tr>
<td>$r_4$</td>
<td>Mean</td>
<td>3.35</td>
<td>3.37</td>
<td>3.13</td>
<td>2.59</td>
<td>2.63</td>
<td>3.23</td>
</tr>
<tr>
<td></td>
<td>Median</td>
<td>3.36</td>
<td>3.33</td>
<td>3.01</td>
<td>2.52</td>
<td>2.55</td>
<td>3.13</td>
</tr>
<tr>
<td>$r_5$</td>
<td>Mean</td>
<td>3.35</td>
<td>3.37</td>
<td>3.13</td>
<td>2.59</td>
<td>2.63</td>
<td>3.23</td>
</tr>
<tr>
<td></td>
<td>Median</td>
<td>3.36</td>
<td>3.35</td>
<td>3.02</td>
<td>2.52</td>
<td>2.55</td>
<td>3.13</td>
</tr>
</tbody>
</table>

**Notes:** The table reports estimates of the mean and the median of the Hill tail-index estimators $\hat{\alpha}_k$ for $k\in\{2.5, 5, 10\}$ percent of the smallest (left-tail) and largest (right-tail) returns for the five assets. Computations are based on 1,000 time series, each containing 5,000 observations.

### Table IV.
Autocorrelations for raw and absolute returns of the asset markets

| Asset | Mean/Median | AC$_1^r$ | AC$_2^r$ | AC$_3^r$ | AC$_{2.5|}|$ | AC$_{5|}|$ | AC$_{10|}|$ |
|-------|-------------|----------|----------|----------|------------|----------|------------|
| $r_1$ | Mean        | 0.02     | -0.01    | -0.01    | 0.10       | 0.01     | 0.01       |
|       | Median      | 0.02     | -0.01    | -0.01    | 0.11       | 0.01     | 0.01       |
| $r_2$ | Mean        | 0.02     | -0.01    | -0.01    | 0.10       | 0.02     | 0.01       |
|       | Median      | 0.02     | -0.01    | -0.01    | 0.11       | 0.02     | 0.01       |
| $r_3$ | Mean        | 0.00     | 0.00     | 0.00     | 0.11       | 0.02     | 0.01       |
|       | Median      | 0.00     | 0.00     | 0.00     | 0.12       | 0.02     | 0.01       |
| $r_4$ | Mean        | 0.02     | -0.01    | -0.01    | 0.11       | 0.02     | 0.01       |
|       | Median      | 0.02     | -0.01    | -0.01    | 0.11       | 0.02     | 0.01       |
| $r_5$ | Mean        | 0.02     | -0.01    | -0.01    | 0.11       | 0.02     | 0.01       |
|       | Median      | 0.02     | -0.01    | -0.01    | 0.11       | 0.02     | 0.01       |

**Notes:** The table displays the estimates of the mean and the median of the autocorrelation of raw returns, AC$_{1|}|$, for lags $\ell\{1, 2, 3\}$, and the autocorrelation of absolute returns, AC$_{2.5|}|$, for lags $\ell\{1, 20, 50, 100\}$. Computations are based on 1,000 time series, each containing 5,000 observations.

### Table V.
Scaling exponent for raw and absolute returns of the five assets

<table>
<thead>
<tr>
<th>$H_r$</th>
<th>$r_1$</th>
<th>$H_{r_1}$</th>
<th>$r_2$</th>
<th>$H_{r_2}$</th>
<th>$r_3$</th>
<th>$H_{r_3}$</th>
<th>$r_4$</th>
<th>$H_{r_4}$</th>
<th>$r_5$</th>
<th>$H_{r_5}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>0.42</td>
<td>0.63</td>
<td>0.42</td>
<td>0.63</td>
<td>0.42</td>
<td>0.63</td>
<td>0.42</td>
<td>0.63</td>
<td>0.42</td>
<td>0.63</td>
</tr>
<tr>
<td>Median</td>
<td>0.42</td>
<td>0.63</td>
<td>0.42</td>
<td>0.64</td>
<td>0.42</td>
<td>0.63</td>
<td>0.42</td>
<td>0.63</td>
<td>0.42</td>
<td>0.63</td>
</tr>
</tbody>
</table>

**Notes:** Estimates of the mean and the median for the scaling exponent of raw returns, $H_r$, and the scaling exponent of absolute returns, $H_{r\nu}$, Computations are based on 1,000 time series, each containing 5,000 observations.

References


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